Negative Binomial Regression in Overcoming Overdispersion in Extreme Poverty Data in Indonesia

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ABSTRACT

Indonesia's extreme poverty status in 2021 was recorded to be high at 4% or 10.86 million people. One of the efforts in poverty alleviation is to analyze the factors influencing extreme poverty. Although the number of studies on poverty in Indonesia continues to grow, the findings are inconclusive because they are often discussed qualitatively. This study aimed to analyze the factors that influence extreme poverty in Indonesia using negative binomial regression. The data used was the amount of extreme poverty in 34 provinces of Indonesia as the response variable. Then, the explanatory variables used consist of 8 from the Central Bureau of Statistics. The analysis stage sought data exploration, the correlation between variables, Poisson regression model specification and assumption test, handling overdispersion with negative binomial regression, and model feasibility test. Based on the AIC value and dispersion ratio, the negative binomial model obtained an AIC value of 920.03 with a dispersion ratio 1.372. It shows that the negative binomial regression model is good enough to model extreme poverty in Indonesia. Furthermore, the factors significantly influencing extreme poverty in Indonesia are households with proper drinking water, housing status, and families with access to appropriate sanitation.

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1. Introduction

One of the complex problems faced by Indonesia is poverty. Poverty is a loss of welfare (deprivation of well-being) [1]. Poverty is not only related to economic conditions but is integrated with social, cultural, religious, political, and other aspects. Indonesia’s poverty rate in March 2021 was 10.14% [2]. This figure increased when compared to the March 2020 period, which amounted to 9.78% [3]. In addition to the high poverty rate, Indonesia in 2021 faces a fairly high extreme poverty status, where the excessive poverty rate in Indonesia is 4% or 10.86 million people [4]. Extreme poverty is where people’s welfare is below the extreme poverty line, equivalent to USD 1.9 PPP (purchasing power parity) [5]. Based on [6], people with an income below the poverty standard are called poor. Similarly, people with incomes below the extreme poverty standard are called the extreme poor.

Extreme poverty is measured using an "absolute poverty measure” consistent across countries and over time. The case of poverty in Indonesia is one of the problems that the government is focusing on so that it can be reduced over time. It aligns with the President’s direction in the Limited Meeting on the Strategy for Accelerating Poverty Reduction on March 4, 2021, so that extreme poverty by 2024 can reach 0% [5].

Several studies have been conducted to analyze poverty and poverty rates in Indonesia. [7] analyzed poverty levels in Indonesia using cluster analysis. [8] used multiple linear regression in analyzing the factors that influence poverty in Indonesia, while [8] used three approaches in exploring the dynamics of poverty in Indonesia. Research on poverty in Indonesia has mostly been studied qualitatively. In terms of quantitative, the study of poverty is dominated by the scope of clustering, and it is still very rare to model and analyze the factors that cause poverty.

The population of extreme poverty is enumerated data, so it is assumed to be Poisson distributed. More than multiple linear regression is required to analyze data structures where the response variable is Poisson distributed. Poisson regression is a good enough model to model this kind of data structure. In real cases in the field, Poisson-distributed data often has a variance value greater than the average, called overdispersion. Violating the equidispersion assumption results in Poisson regression needing to be better to model count data with overdispersion. To overcome this problem several models can overcome the problem of overdispersion and its applications, including generalized Poisson regression [9], quasi poisson regression, zero inflated poisson [10] and binomial negative regression.


Based on the above problems, an analysis is needed to model and determine the factors that affect extreme poverty in Indonesia. This research is expected to be an additional insight for readers to analyze the factors that cause extreme poverty as a consideration for eradicating the problem of extreme poverty in Indonesia.

2. Research Methods

2.1 Data Description

The data used in this study are secondary: data sourced from the deputy assistant for poverty management of the Indonesian Ministry of Coordination, Human Development, and Culture. In this study, data on the number of Indonesia's extremely poor population in 2021 was used. The data consists of 34 provinces in Indonesia with 8 independent variables.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y )</td>
<td>Number of extreme poor in Indonesia</td>
</tr>
<tr>
<td>( X_1 )</td>
<td>Percentage of Indonesia's open unemployment rate</td>
</tr>
<tr>
<td>( X_2 )</td>
<td>Percentage of school enrollment rate (16-18 years)</td>
</tr>
<tr>
<td>( X_3 )</td>
<td>Percentage of school enrollment rate (19-24 years)</td>
</tr>
<tr>
<td>( X_4 )</td>
<td>Human development index</td>
</tr>
<tr>
<td>( X_5 )</td>
<td>Percentage of households with improved drinking water sources</td>
</tr>
<tr>
<td>( X_6 )</td>
<td>Percentage of households with PLN lighting source</td>
</tr>
<tr>
<td>( X_7 )</td>
<td>Percentage of households with own home ownership status</td>
</tr>
<tr>
<td>( X_8 )</td>
<td>Percentage of households with access to proper sanitation</td>
</tr>
</tbody>
</table>

Data source: ([15],[16])
2.2 Data Analysis Procedure

The data analysis procedure in this study is as follows:

1. Data exploration
2. Creation of a correlation matrix to see the correlation between explanatory variables.
3. Poisson regression model formation

   Poisson regression model is a nonlinear model often used to model count data. The general form if the discrete random variable \( y \) is a Poisson distribution with parameters \( \mu_i \) is the average number of events that occur in a certain time interval. The general model of Poisson regression is: [17]
   \[
   \mu_i = t_i \exp(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \ldots + \beta_p X_{pi})
   \]

4. Multicollinearity check of Poisson regression model

   Checking the multicollinearity of the model can be seen through the Variance Inflation Factor (VIF) with an indication of multicollinearity if the VIF value > 10 and Tolerance > 0.10 with the following calculation formulation: [18]
   \[
   VIF = \frac{1}{1-R_i^2}
   \]
   with \( R_i^2 \): coefficient of determination

5. Overdispersion check

   According to [19], Poisson regression is said to be overdispersed if the variance value is greater than the mean value. If there is overdispersion in discrete data and still using Poisson regression as a solution method, an invalid conclusion will be obtained because the standard error value is underestimated.

6. Formation of negative binomial regression model

   The Negative Binomial regression model can be used to model Poisson data that experiences overdispersion because the Negative Binomial distribution is an extension of the Poisson-Gamma distribution that contains a dispersion parameter \( \theta \). The Negative Binomial regression model is expressed as follows: [17]
   \[
   \hat{y}_i = t_i \exp(\hat{\beta}_0 + \hat{\beta}_1 X_{1i} + \hat{\beta}_2 X_{2i} + \ldots + \hat{\beta}_p X_{pi})
   \]

7. Parameter estimation of Negative Poisson Binomial Regression

   Parameter estimation uses the Maximum Likelihood Estimation (MLE) method with the Newton-Raphson iteration approach to maximize the likelihood function. The likelihood function of Negative Binomial regression is as follows: [20]
   \[
   L(y; \mu, \theta) = \prod_{i=1}^{n} \frac{i^{y_i} \Gamma(y_i + \frac{1}{\theta})}{\Gamma(y_i) \Gamma(\frac{1}{\theta})} \left( \frac{\theta}{1+\theta} \right)^{y_i} \left( \frac{1}{1+\theta} \right)^{\frac{1}{\theta}}
   \]

8. Parameter significance test

   Simultaneous significance testing for parameter estimation of the Negative Binomial Regression model uses the devians test with the following hypothesis:
   \( H_0: \beta_k = 0 \)
   \( H_1: \beta_k \neq 0; k=1,2,\ldots,p \)

   Test statistics:
   \[
   W_k = \frac{\hat{\beta}_k}{SE(\hat{\beta}_k)}
   \]

   \( H_0 \) is rejected if the Wald test statistic or \( t_{\text{count}} > t_{(n-k, \frac{\alpha}{2})} \), meaning that the \( k \)-th parameter is significant to the negative binomial regression model.

9. Model fit test

   This test uses the Pseudo value \( R^2 \)
   \[
   R^2_p = 1 - \frac{L_p}{L_i}
   \]
   where
   \( L_p \): likelihood function value of the complete model
   \( L_i \): the likelihood function value of the model containing only the intercept.
3. Result and Discussion

3.1. Data Exploration

The distribution of the number of poor people in Indonesia across the 34 provinces of Indonesia is presented in Figure 1.

**Figure 1.** Histogram of the Number of Extremely Poor People in Indonesia in 2021

Figure 1 shows that the provinces with the highest number of extremely poor in Indonesia are East Java, with 1,730,709 people; West Java, with 1,659,977 people; and Central Java, with 1,620,269 people. Meanwhile, the provinces with the least extremely poor people in Indonesia are Bangka Belitung Islands, with 18,857 people; North Kalimantan, with 19,538 people; and North Maluku, with 26,253 people.

**Figure 2.** Scatterplot of matrix

Figure 2 shows a scatterplot of the data matrix to see the relationship pattern between variables. The result is that the independent variables do not have a good or close linear relationship.

3.2. Correlation Matrix

A correlation matrix is a table that shows the correlation coefficients between variables. Each cell in the table shows the correlation between two variables. The criterion for the calculation results if the correlation between variables is stronger if it is close to the value of 1 (perfect positive correlation) and -1 (perfect negative correlation) [21].
In the correlation matrix table above, it can be seen that the correlation between variables varies greatly. A very strong correlation is between the variables $X_6$, i.e., households with electric lighting from PLN, and $X_8$ households with access to proper sanitation of 0.78. Meanwhile, the variables with a weak correlation are between variables $X_4$, Human Development Index, and $Y$, namely the number of extremely poor people in Indonesia, amounting to -0.04.

### 3.3. Poisson Regression Model

#### Table 3. Poisson Regression Test Results

| Variable | Estimate | Std.Error | Z-value | Pr(>|z|) |
|----------|----------|-----------|---------|----------|
| Intercept| -8.79    | 1.65      | -533.6  | <2e-16   | ***    |
| $X_1$    | 8.21     | 2.71      | 302.8   | <2e-16   | ***    |
| $X_2$    | 1.39     | 1.05      | 133.1   | <2e-16   | ***    |
| $X_3$    | -4.10    | 9.01      | -455.2  | <2e-16   | ***    |
| $X_4$    | 1.86     | 2.46      | 755.4   | <2e-16   | ***    |
| $X_5$    | 8.92     | 7.44      | 1199.0  | <2e-16   | ***    |
| $X_6$    | -1.63    | 6.80      | -239.5  | <2e-16   | ***    |
| $X_7$    | 1.10     | 7.22      | 1520.3  | <2e-16   | ***    |
| $X_8$    | -9.27    | 7.42      | -1249.8 | <2e-16   | ***    |

The Poisson regression model obtained can be written as follows:

$$Y = t \cdot \exp(8.79 + 8.21 X_1 + 1.39 X_2 - 4.10 X_3 + 1.86 X_4 + 8.92 X_5 - 1.10 X_6 + 1.10 X_7 - 9.27 X_8)$$

Based on the model above, a model is obtained in which all variables significantly influence poverty. Furthermore, from this model, multicollinearity and equidispersion assumptions are checked.

### 3.4. Multicollinearity Check of Poisson Regression

#### Table 4. Results of VIF and Tolerance of Poisson Regression

<table>
<thead>
<tr>
<th>Variables</th>
<th>VIF</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>3.73</td>
<td>0.27</td>
</tr>
<tr>
<td>$X_2$</td>
<td>3.33</td>
<td>0.3</td>
</tr>
<tr>
<td>$X_3$</td>
<td>2.33</td>
<td>0.43</td>
</tr>
<tr>
<td>$X_4$</td>
<td>6.81</td>
<td>0.15</td>
</tr>
<tr>
<td>$X_5$</td>
<td>3.39</td>
<td>0.29</td>
</tr>
<tr>
<td>$X_6$</td>
<td>7.14</td>
<td>0.14</td>
</tr>
<tr>
<td>$X_7$</td>
<td>2.04</td>
<td>0.49</td>
</tr>
<tr>
<td>$X_8$</td>
<td>5.98</td>
<td>0.17</td>
</tr>
</tbody>
</table>

In the results of Table 4, it can be seen that the Tolerance values of all independent variables are > 0.10, while the VIF value of each variable is < 10. Based on both results, it can be said that $H_0$ is accepted with the conclusion that there is no multicollinearity between independent variables.
3.5. The Checking of Poisson Regression Overdispersion

The checking of the Poisson regression assumption is data equidispersion in Table 5 in which the data will experience equidispersion if the estimated value of the dispersion ratio, Pearson chi-square = 1, and does not experience overdispersion if the p-value > 0.05.

<table>
<thead>
<tr>
<th>Dispersion ratio</th>
<th>197.209,936</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson’s Chi-Squared</td>
<td>4.930,248,4</td>
</tr>
<tr>
<td>P-value</td>
<td>&lt;0,001</td>
</tr>
<tr>
<td>AIC</td>
<td>4.245,071</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0,79</td>
</tr>
</tbody>
</table>

Table 5 shows that the data experienced overdispersion, and the Poisson regression model is not good enough to model poverty data. The existence of overdispersion causes the Poisson regression model to be less good. It is due to the high error rate, so one way that can be done to overcome the overdispersion case in Poisson regression is to replace the Poisson distribution assumption with a Negative Binomial distribution.

3.6. Negative Binomial Regression Model

The negative binomial regression model can be written as follows:

$$Y_i = t \exp(-0.59 + 0.12X_1 + 0.00X_2 + 0.07X_4 - 0.09X_5 + 0.07X_7 - 0.08X_8)$$

The interpretation of the negative binomial regression model obtained is as follows:

1. Coefficient $X_1$ is the percentage of the open unemployment rate. That every 1% increase in the open unemployment rate of a province will cause the average number of extremely poor people in a province to be $\exp(0.12) = 1.13$ times the initial value with the assumption that other variables are considered constant. Based on the results of this study, it can be said that $X_1$ has no significant effect on the extreme poverty rate in Indonesia. This result is supported by research conducted by [22] with a research focus on the open unemployment rate on poverty in Aceh Province.

2. Variable $X_2$ Percentage of school enrollment rate (16 - 18) years. Research [23] revealed that the higher the school enrollment rate, the lower the poverty rate in Bangka Belitung Province. It is directly proportional to the results of this study where although it does not have a significant effect, every 1% increase in the school enrollment rate of a province will cause the average number of extremely poor people in a province to be $\exp(0.00) = 1$ times the initial value with the assumption that other variables are considered constant.

3. Coefficient $X_3$ is the percentage of school enrollment rate (19-24) years where every 1% increase in the school enrollment rate of a province will cause the average number of extremely poor people in a province to be $\exp(-0.01) = 0.99$ times the initial value with the assumption that other variables are considered constant. This result is supported by research conducted by [24] that a negative relationship exists between school enrollment and poverty.

4. In the context of variable $X_4$, the research on Human Development Index (HDI) variable has been carried out by [25], where HDI influences poverty. The result of this study is that every 1% increase in the human development index (19 - 24) years in a province will cause the average number of extremely poor people in a province to be $\exp(0.09) = 1.09$ times the initial value with the assumption that other variables are considered constant.
5. Research on the effect of decent drinking water and poverty in East Java has been conducted by [26], where poverty impacts community access to clean water. This study's results align with this study, where the percentage of households with decent drinking water ($X_5$) correlates to extreme poverty. Every 1% increase in households with proper drinking water in a province will cause the average number of extremely poor people to $\exp(0.07) = 1.07$ times the initial value with the assumption that other variables are considered constant. Based on the $p$-value < 0.05, $X_5$ has a significant effect.

6. The coefficient of the percentage of households with electricity from PLN ($X_6$) shows that every 1% increase in households with electricity from PLN in a province will cause the average number of extreme poor people in the province to $\exp(-0.00) = 0.99$ times the initial value, assuming other variables are considered constant.

7. Coefficient $X_7$ is the coefficient of the percentage of households with their house status. Every 1% increase in households with own house status in a province will cause the average number of extreme poor people in the province to $\exp(0.07) = 1.07$ times the initial value, assuming other variables are considered constant.

8. Access to proper sanitation in the community can indirectly reduce poverty levels [27]. In this study, it is said that every 1% increase in households with access to proper sanitation in a province will cause the average number of extreme poor people to $\exp(-0.07) = 0.93$ times the initial value, assuming other variables are considered constant.

3.7. Multicollinearity Check of Negative Binomial Regression

Table 7. Results of VIF and Tolerance of Negative Binomial Regression

<table>
<thead>
<tr>
<th>Variables</th>
<th>VIF</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>1.95</td>
<td>0.51</td>
</tr>
<tr>
<td>$X_2$</td>
<td>2.34</td>
<td>0.43</td>
</tr>
<tr>
<td>$X_3$</td>
<td>2.34</td>
<td>0.43</td>
</tr>
<tr>
<td>$X_4$</td>
<td>5.46</td>
<td>0.18</td>
</tr>
<tr>
<td>$X_5$</td>
<td>1.87</td>
<td>0.53</td>
</tr>
<tr>
<td>$X_6$</td>
<td>3.66</td>
<td>0.27</td>
</tr>
<tr>
<td>$X_7$</td>
<td>2.48</td>
<td>0.4</td>
</tr>
<tr>
<td>$X_8$</td>
<td>3.85</td>
<td>0.26</td>
</tr>
</tbody>
</table>

In the results of Table 4. It can be seen that the Tolerance values of the dependent variables > 0.10, while the VIF value of each variable is <10. Based on both results, it can be said that $H_0$ is accepted that there is no relationship between independent variables.

3.8. The Checking of Negative Binomial Regression Overdispersion

Table 8. The Checking Calculation of Negative Binomial Regression Overdispersion

<table>
<thead>
<tr>
<th>Dispersion ratio</th>
<th>1.372</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pearson's Chi Squared</td>
<td>34.29</td>
</tr>
<tr>
<td>P-value</td>
<td>0.102</td>
</tr>
<tr>
<td>AIC</td>
<td>920.03</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.79</td>
</tr>
</tbody>
</table>

In Table 7, we can see that the value of the dispersion ratio and Pearson's chi squared in the negative binomial model is smaller than the Poisson model, namely 1.371 and 34.29. Then, for the p-value generated in the negative binomial model, which is > 0.05, this indicates that the data is no longer experiencing overdispersion. In addition, when viewed from the value of Pseudo $R^2$ value, negative binomial regression is 0.79. It can be concluded that 79% of the model used can be good enough to describe or explain the case of extreme poverty in Indonesia, while 21% is influenced by other factors not included in the model.

4. Conclusions

The negative binomial Poisson regression model is good enough to model poverty data in Indonesia. This is because the data are overdispersed, so it needs to be overcome by negative binomial Poisson regression. Then, this can be seen from the AIC value that negative binomial poisson regression has a smaller AIC value than poisson regression, which
The value of $R^2$ in negative binomial regression is 0.79 with a p-value of 0.102 greater than 0.05, so it is said that handling overdispersion with negative binomial regression can be resolved. The resulting negative binomial regression model is:

$$Y_i = \exp(-0.59 + 0.07X_1 + 0.07X_2 - 0.07X_3)$$

Furthermore, the factors that significantly influence extreme poverty in Indonesia are households with proper drinking water, their own housing status, and households with access to proper sanitation.

**Acknowledgment**

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**References**


