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Flower Pollination Algorithm for Vehicle Routing Problem with Time Windows (VRPTW)

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Abstract: *Vehicle Routing Problem with Time Windows* (VRPTW) is a vehicle route deciding problem that is used in order to serve customer who involved more than one vehicle with a limited time, so as a minimum distance route is obtained without disobeying vehicle capacity cargo restriction and time range. *Flower Pollination Algorithm* (FPA) is an algorithm which inspires from nature and that is flower pollination process toward a plant. Within an FPA, there are two main steps to use, they are global flower pollination and local flower pollination. Those two steps are determined by using switch probability parameter. This program is made in Java language program to apply FPA in solving VRPTW which is implemented in three example cases, they are small-scale datum with 25 customers, medium-scale datum with 50 customers, and big-scale datum with 100 customers. According to the results, it can be concluded that the larger number of flowers and iterations can affect the number of total minimum travel distance become smaller. Furthermore, a better total minimum travel distances also will be obtained if the value of switch probability parameter is larger.

Keywords: Flower Pollination Algorithm, Vehicle Routing Problem, Time Windows, Parameters.

1. Introduction

In nowadays era many companies compete in selling products. Every company has innovation to develop its product in order to please the consumer. In a product selling is always started by production process so that the product is ready for sale, but before the product reaches the consumer, there is a process of product distribution. Based on **[1]**, distribution is a dispatch process (sharing, sending) good to some people or to some places. However, according to **[2]** distribution is a distribution of goods from one place to another from producer to consumer to be utilized.

Every company has its own way to get the optimal profit. One way to optimize profit is by minimizing the distribution costs. Distribution costs can be minimized by minimizing the distance of distribution. The distribution costs problems are included in *vehicle routing problem* (VRP). VRP is a problem within a distribution system that has a purpose to make a route with the shortest distance, with a group of vehicle whose capacity is known, in order to fulfil the consumer's willing with location and number of request that are known **[3]**. VRP was first time introduced by Dantzig and Ramzer in 1959. In process of distribution, not all consumers can receive goods at any time, but only in certain time. Those case lead a new problem which is *Vehicle Routing Problem with time windows* (VRPTW), where VRPTW is a development problem from VRP with time constraint addition.

Vehicle Routing Problem with Time Windows (VRPTW) is a problem that consists of some vehicles, capacity and in certain time windows. In this case, *time windows* can be interpreted as the time span $[a_i, b_i]$ which is needed for every i consumer in order to get service. A vehicle is allowed to come before a_i and wait until the consumer can approve the service. In waiting time process there is no service process that is carried out, service process is done when a_i is started. The upcoming vehicle after b_i is not allowed (with i is a customers index **[4]**.

In solving *Vehicle Routing Problem with Time Window* (VRPTW), there are several algorithms that have been used, among others, *Particle Swarm Optimization* **[5]**, Simulated Annealing **[6]**. Currently, there are number new algorithms one of which is Flower Pollination Algorithm (FPA). *Flower Pollination Algorithm* (FPA) was the first time discovered by Xin She Yang in 2012. *Flower Pollination Algorithm* (FPA) adopts from flower pollination. In this case, pollination process consists of two types which are local pollination and global pollination **[7]**. The implementation of *Flower Pollination Algorithm* (FPA) is still relatively small in solving the optimization problems. Based on Yang (2012) in his journal entitled *"Flower Pollination Algorithm for Global Optimization*" said that the result of FPA simulation is more efficient than *Genetic Algorithm* (GA) and *Particle Swarm Optimization* (PSO) **[7]**.

Based on the previous explanation, this paper is organized become six sections. The next section is an overview about VRPTW. Section 3 discusses about FPA which is introduced by Yang (2012) **[7]**. The steps of implementation is discussed in section 4, while the discussion and result is presented in section 5. The last section is Conclusion.

2. Model Formulation

Vehicle Routing Problem with Time Windows is VRP problem with additional time, where service from each costumer starts within certain time interval **[8]**. Time windows is defined as time span $[a_i, b_i]$ which is needed by a costumer to obtain service. A vehicle is allowed to arrive before a_i, and wait untill that customer is available, however the arrival after b_i is not permitted **[4]**. The aim of VRPTW is minimizing the total cost of travel without ignoring vehicle capacity restriction and time windows depots. The route formation is carried out so that each costumer is only visited once by one vehicle, and each vehicle starts and ends its route at the depots **[9]**.

According to **[9]** mathematical model from VRPTW is as following:

$$Min\ Z = \sum_{k=1}^{K} \sum_{i=1}^{L} \sum_{j=1}^{L} c_{ij} x_{ijk}$$

With some constraints:

1. Total of request from each city in a route which is passed every vehicle is not more than the capacity of vehicle

$$\sum_{i=1}^{L} g_i y_{ik} \leq q \ \forall\ k = 1,2, \dots, K \qquad (1)$$

2. Every costumer is exactly visited one time by one vehicle

$$\sum_{k=1}^{K} y_{ik} = 1, \quad \forall\ i = 1, 2, \dots, L \qquad (2)$$

3. Make sure every route that is passed through is complete.

$$\sum_{i=1}^{L} x_{ijk} = y_{jk,} \forall\, k, j, \quad k = 1,2,\dots,K, \qquad j = 1,2,\dots,L \qquad (3)$$

$$\sum_{k=1}^{L} x_{ijk} = y_{ik,} \forall\, k, i, \quad k = 1,2,\dots,K, \qquad i = 1,2,\dots,L \qquad (4)$$

$$\sum_{i,j} \sum_{\in S\,x\,S} x_{ijk} \leq |s| - 1, s \subset \{1,2,\dots,L\}, \forall\, k = 1,2,\dots,L \quad (5)$$

4. Make sure that two close vehicles are lied on the same route.

$$sl_i + t_{ij} \leq s_{ej}, \forall\, k, i, j, \qquad k = 1,2\dots,K, \qquad i = 1,2,\dots,L,$$
$$j = 1,2,\dots,L \qquad (6)$$

5. Service restrictions toward the costumer (time windows) :

$$e_i \leq se_i \leq s_i \;\forall\, i = 1,2,\dots,L \qquad (7)$$

$$x_{ijk}, y_{ik} \in (0,1) \;\forall\, k, i, j, k = 1,2,\dots,K, i = 1,2,\dots,L, j = 1,2,\dots,L$$

where:

$$x_{ijk} = \begin{cases} 1, k's\ directly\ vehicle\ request\ from\ i\ to\ j \\ \quad 0, \quad if\ it\ is\ not\ such\ way \end{cases}$$

$$y_{ik} = \begin{cases} 1, i's\ request\ customer\ is\ fulfilled\ by\ k\ vehicle \\ \quad 0, \quad if\ it\ is\ not\ such\ way \end{cases}$$

x_{ijk}	: Vehicle k that takes care of costumer i after visiting costumer j
c_{ij}	: Distance cost from costumer i to costumer j (can be interpreted as distance)
q	: Vehicle capacity
g_i	: Request from costumer i
$[e_i, l_i]$	: *Time windows* toward costumer i
e_i	: The most early time to do service to costumer i
l_i	: The most end time to do service to costumer i
se_i	: Time when service in costumer's node i is started
sl_i	: Time when service in costumer's node i end
s_i	: Total amount of time service from customer's node i
t_{ij}	: Time travelling from customer's node i to customer's node j (in this case, is exampled that $t_{ij} = c_{ij}$)
i	: Early city index
j	: Destination city index
K	: The amount of vehicle
L	: The amount of customer
S	: Total number of customer

3. Flower Pollination Algorithm (FPA)

Flower Pollination Algorithm (FPA) was introduced first time by **Xin She Yang** in 2012 **[10]**. Flower Pollination Algorithm (FPA) is an algorithm which inspires from nature especially flowery plant pollination. This algorithm is adjusted with the exact flower pollination that is pollination concept which consist of two types, they are global pollination and local pollination. There are four approaches that are used in this study. Those approaches are formulated as follows:

a. Biotic pollination and global pollination is seen as global pollination with pollen carrying pollination organism that carry out movement *Lévy Flights*;
b. Abiotic pollination and self-pollination is seen as local pollination
c. Flower constancy is seen as proportional reproduction opportunity with similarity from two involved flowers.
d. Local pollination and global pollination is settled up by *switch probability* $p \in [0, 1]$.

Two key steps which are used in *Flower Pollination Algorithm* (FPA) are below:

a) Local pollination

Local pollination is seen as abiotic pollination and self-pollination. Abiotic pollination means pollen transfer to the pistil head that does not require pollinating organisms, but only needs wind assistance and diffusion of water in the soil. On the other hand, self-pollination occurs if pollen falls from different flowers but comes from the same plant. The step of local pollination is done by using the second and third approaches which can depict mathematically as below:

$$x_i^{t+1} = x_i^t + \epsilon\left(x_j^t - x_k^t\right) \tag{8}$$

With information
x_i^t : $i -$ flower to t-iteration
x_i^{t+1} : $i - flower\ to\ t + 1$-*iteration*
ε : real random numbers that uniformly distributed between 0 and 1
j, k : index of two *d*ifferent flowers which still place in one plant

b) Global Pollination

Global pollination is seen as biotic pollination and cross pollination. Biotic pollination means that the transfer of pollen to the stigma is carried out by pollinating organisms such as bees, bats, birds and flies. Whereas cross pollination occurs when pollen falls from flowers from different plants, so that in the process of global pollination, pollinating organisms can fly in long distance and do *Lèvy Flights* movement based on *Lèvy* distribution. This process can be presented mathematically as follows:

$$x_i^{t+1} = x_i^t + \gamma L(\lambda)(g_* - x_i^t) \tag{9}$$

where
x_i^t : $i -$ flower to t-iteration
x_i^{t+1}: $i -$ flower to $t + 1$-iteration
$L(\lambda)$: *stepsize* or flying distance that is based on *Lévy* distribution with $L > 0$
g_* : The best flower element in the iteration to t

With γ is a *stepsize* controller parameter at intervals with $\gamma > 0$ **[5]**, and λ is a parameter at interval [1,2] **[11]**. The *Lèvy* distribution is represented by the following equation (6),

$$L \sim \frac{\lambda A(\lambda)\sin(\pi.\lambda/2)}{\pi}\frac{1}{s^{1+\lambda}} \tag{10}$$

where $\Gamma(\lambda)$ is a gamma function, and the distribution is applied to *step length* > 0. According **[12]**, *step length* (s) could be calculated using a normal distribution with the following formula,

$$s = \frac{U}{|v|^{\frac{1}{\lambda}}} \tag{11}$$

where variable U is a random real number that is normally distributed with mean 0 and variance σ^2. V is a real random number that is normally distributed with a mean of 0 and variance σ_v^2. According to **[10]**, the value of U can be approximated by a real random number which is normally distributed N (0,1) multiplied by σ. And the value of σ_v^2 is equal to 1 so the value of V is a real random number that is normally distributed $N \sim (0,1)$ too. Equation (11) can be written as:

$$s = \frac{u \times \sigma}{|v|^{\frac{1}{\lambda}}} \quad (12)$$

The value of σ is determined based on

$$\sigma = \left(\frac{A(1+\lambda).\sin(\pi.\lambda/2)}{A\left(\frac{1+\lambda}{2}\right).\lambda.2^{\frac{\lambda-1}{2}}} \right)^{\frac{1}{\lambda}} \quad (13)$$

where Γ is a gamma function. According to **[13]**, the function of $\Gamma: (0, \infty) \to \mathbb{R}$, is a gamma function which is defined as:

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx \quad (14)$$

4. Solving VRPTW using FPA

Methods for solving the research problem are listed below:

a. Defining the objective function $f(x), x = (x_1, x_2, \dots, x_n)^T$.
b. Generating population as much as n flower (pollen gametes) x_j $(j = 1,2, \dots, n)$.
c. Calculating the objective function F_j
d. Finding the best solution (g_*) in the initial population.
e. Generating $rand_i$ randomly, then defining switch probability $p \in [0,1]$.
f. If $rand_i \leq p$ so it will make a global pollination that corresponds with Lévy Flights.
g. If $rand_i > p$ so it will make a local pollination with with ϵ uniform distribution.
h. Obtaning a new flower (x_i) of a local pollination and a global pollination.
i. Evaluating/calculating the objective function F_i.
j. Making a selection by comparing the results of F_i and F_j. If $F_i < F_j$ so F_j becomes a new solution and vice versa.
k. Sorting the flower based on the objective function and determining the best flower (g_*).
l. Determining and saving the best flower.

5. Result and Discussion

The data employed on this problem is the secondary data which is obtained from http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-with-time-windows-instances/. The data consist of three types which are the data from small scale data (25 customers), medium data (50 customers), and big data (100 customers).

The parameter used to solve VRPTW by using data of 25 customers is number of flowers (n) = 10; 50; 100, number of maximum iterations $(maks\_iterasi)$ = 10; 100; 1000, vehicle capacity = 500, number of maximum vehicle = 7, the parameter of controller *stepsize* (α) = 0.01 **[6]**, *switch probability* (p) = 0.25; 0.5; 0.8, and value $\lambda = 1{,}5$ **[11]**. The final results obtained are listed in **Table 1**.

Tabel 1. The Results of The Objective Function

Maximum Iterations	Number of flowers	*p*		
		0.25	**0.5**	**0.8**
10	**10**	859,39	832,06	826,95
	50	789,94	783,08	778,89
	100	777,56	767,72	754,58
100	**10**	749,01	744,19	742,52
	50	726,51	718,53	699,64
	100	600,64	574,50	529,64

1000	**10**	507,33	477,10	457,50
	50	446,93	435,10	417,75
	100	408,66	405,01	**394,28**

The best objective function of 25 customers is 394,28. It could be seen the objective function above, it could be concluded as follows: The higher number of flowers the better quality of the objective. The higher number of iteration the better quality of the objective function. The greater of p value, tends to produce the better solution quality.

The parameter used to solve VRP by using data of 50 customers is number of flowers $(n) = 10; 50; 100$, number of maximum iterations $(maks\_iterasi) = 10; 100; 1000$, vehicle capacity = 1000, number of maximum vehicle = 10, the parameter of controller *stepsize* (α) = 0.01 **[6]**, *switch probability* $(p) = 0.25; 0.5; 0.8$, and value $\lambda = 1{,}5$ **[11]**. The final results obtained are listed in **Table 2**.

Table 2. The Result of The Objective Function

Maximum Iterations	**Number of Flowers**	***p***		
		0.25	**0.5**	**0.8**
10	**10**	2396,24	2383,19	2356,54
	50	2315,28	2306,85	2286,31
	100	2272,24	2180,20	2146,44
100	**10**	2066.46	2024,29	2021,43
	50	2015,66	1989,65	1961,17
	100	1956,26	1935,79	1927,63
1000	**10**	1794,28	1781,86	1735,92
	50	1673,89	1645,07	1617,02
	100	1599,66	1588,42	**1525,59**

The best objective function of 50 customers is 1525,59.As seen the objective function above, it could be concluded as follows : The higher number of flowers the better quality of the objective. The higher number of iteration the better quality of the objective function. The greater of p value, tends to produce the better solution quality.

The parameter used to solve VRPTW by using data of 100 customers is number of flowers $(n) = 10; 50; 100$, number of maximum iterations $(maks\_iterasi) = 10; 100; 1000$, vehicle capacity = 1000, number of maximum vehicle = 20, the parameter of controller *stepsize* (α) = 0.01 **[6]**, *switch probability* $(p) = 0.25; 0.5; 0.8$, and value $\lambda = 1{,}5$ **[11]**. The final results obtained are listed in **Table 3**.

Table 3. The final result of the objective function

Maksimum Iteration	**Number of flowers**	***p***		
		0.25	**0.5**	**0.8**
10	**10**	4444,61	4442,29	4417,30
	50	4408,58	4388,45	4367,60
	100	4343,46	4325,12	4318,39
100	**10**	4267,04	4259,36	4245,30
	50	4225,79	4196,04	4134,54
	100	4109,36	4093,53	4052,31
1000	**10**	3988,39	3985,49	3967,42

	50	3958,17	3949,09	3915,87
	100	3883,20	3809,19	**3781,69**

The best objective function of 100 customers is 3781,69. As seen the objective function above, it could be concluded as follows: The higher number of flowers the better quality of the objective. The higher number of iteration the better quality of the objective function. The greater of p value, tends to produce the better solution quality.

6. Conclusion

The implementation program to solve a case study using different parameters show the results of a wide variety of the objective function value. In connection with the results of the implementation, it reveals that the higher number of flowers and number of maximum iterations influence the quality of the value of the objective function produced. The higher number of flowers and number of maximum iterations the better to produce the solution quality. Correspondingly, the higher value *switch probability* (p) has tendency to produce a better solution.

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