

GROSS PREMIUM VALUATION METHOD IN DETERMINING PREMIUM RESERVES IN LIFE INSURANCE

Rendi Rivaldo^{1*}, Hendra Perdana², Wirda Andani³

^{1,2,3} Department of Statistics, Faculty of Mathematics and Natural Sciences, Universitas Tanjungpura Jl. Prof. Dr. H. Hadari Nawawi, Pontianak, 78124, West Kalimantan, Indonesia

Corresponding Author's E-mail: h1091201028@student.untan.ac.id

Abstract: Life insurance companies maintain reserve funds to pay insurance policy claims, known as premium reserves. Premium reserves are calculated using two approaches: retrospective and prospective. The prospective approach involves calculating the present value of all future expenses minus the total future income for each policyholder, using the Gross Premium Valuation (GPV) method. The GPV method takes into account initial costs, maintenance costs, and administration costs. The case study results indicate that the premium reserve using the GPV method starts at zero in the first year, increases until the last payment year, and then decreases after the payment period until the end of the coverage period. For policyholders of different genders but the same age, the premium reserve for men is greater than for women. Additionally, for male policyholders of varying ages, the premium reserves required increase with age. Furthermore, for male policyholders of the same age but with different interest rates, a higher interest rate results in a smaller premium reserve requirement.

Keywords: Prospective Reserves, GPV, Gross Premiums

INTRODUCTION 1.

Humans will certainly experience unexpected events, such as illness, accidents, loss of property, and others. These events can happen anytime, anywhere, and, of course, they will certainly cause personal losses, both financially and mentally. Humans need a guarantee to face events like this and reduce the risk of loss. One of the guarantees that can be obtained is by participating in an insurance program. Education, health, life, and so on are some of the many types of insurance. Life insurance is an agreement between the insurance company and the insured in case of a risk of death to the insured, and it will provide death benefits to the insured's heirs with a certain amount [1]. There are several types of life insurance based on the length of the coverage period and the type of policy offered by the insurance company, one of which is term life insurance.

This type of insurance provides protection coverage within a certain period, and if death occurs within the specified period, the company will provide insurance benefits to the insured party [2]. When a person takes out insurance, he and the company agree to a written contract designed by the company and regulatory authorities. This contract is called an insurance policy. The policy stipulates that the policyholder pays a certain amount of money, called a premium, and the company will pay a certain amount if something happens to the policyholder [3].

Insurance premiums are the amount the insured must pay to the insurance company based on the agreed policy and then paid back by the insurance company based on the chosen policy [4]. Insurance premiums are divided into two types, namely net premiums and gross premiums. Gross premium is the net premium plus commission (additional costs) the insurance company receives [5]. Insurers bear three costs: initial costs, maintenance costs, and administrative costs. Initial costs are costs incurred by insurance companies when issuing insurance contracts. In contrast, insurance companies use maintenance costs to manage premiums as securities at the Custodian Bank, and Administrative Costs are those used by insurance companies for operational services to the insured [6].

In insurance management, an insurance company reserves a certain amount of money that will be used to pay insurance policy claims, which are called premium reserves. Premium reserves are obtained from the



215

difference between the cash value of payments and the compensation value at the time of coverage [7]. According to Hasnah, insurance companies must have premium reserves so they do not have difficulty paying premiums when policy claims occur [8]. Premium reserves are divided into two categories: prospective reserves, calculated based on future values, and retrospective reserves, calculated based on past values [9]. One method of calculating premium reserves with a prospective approach is the Gross Premium Valuation method (GPV).

The GPV method is one of the premium reserve calculation methods based on the gross premium value. In addition, this method also considers other costs, such as initial costs, administrative costs, maintenance costs, etc., so that the calculation results are considered more accurate than other premium reserve methods. The GPV method can be calculated using a prospective approach based on future values or a retrospective approach based on past values. In this study, the calculation will use a prospective approach. Premium reserves using the GPV method can be calculated by subtracting the present value of all cash outflow components from the present value of all cash inflow components [10].

Research on premium reserves and the GPV method has been conducted by several researchers before, including research on the topic of premium reserves on joint life insurance with the Canadian method, which produces an inversely proportional relationship between age and premium reserve value, as well as interest rates and premium reserve value [11]. Another study on prospective premium reserves with the GPV method in endowment life insurance concluded that the reserve value at the beginning of the first year is zero and will also obtain reserves with a smaller value and gross premiums for larger interest rates [12].

In this research, the type of insurance used is term life insurance, and the interest rate used is BI-7 Days Reverse Repo Rate (BI7DRR). In addition, the premium reserve calculation will be compared with several conditions. These conditions include differences in the gender of the insured at the same age, differences in the insured's age with the gender of the insured, namely male, and differences in interest rates used with the same age and gender of the insured. Based on the background and the results of previous studies on the GPV method, this study aims to obtain the formula and results of the calculation of term life insurance premium reserves using the GPV method for several insured conditions.

2. METHODOLOGY

2.1. Mortality Table

A mortality table is a list of deaths and lives of a group of people [13]. The probability that an insured who is now x years old for the next one year will remain alive can be expressed by:

$$p_x = \frac{l_{x+1}}{l_x} \tag{1}$$

The probability that an insured who is now x years old will live to be x + n years old can be expressed by:

$$_{n}p_{x} = \frac{l_{x+n}}{l_{x}} \tag{2}$$

The probability that the insured who is now *x* years old and will die during the next one year is expressed by:

$$q_x = \frac{l_x - l_{x-1}}{l_x}$$
(3)

Subhan also stated that the probability that the insured, who is now x years old, will die before the age of x + n years can be denoted and expressed by [13]:

$$_{n}q_{x} = 1 - \frac{l_{x+n}}{l_{x}} \tag{4}$$

2.2. Discount Factor

An investment of one will accumulate to 1 + i at the end of the investment period. This 1 + i value is called the accumulation factor because it accumulates the investment value from the beginning to the end of the period. Often customers are confused about determining the initial value of investment so that the value is fixed when the period ends. If a customer invests one at the beginning of the period and wants the investment value to remain one at the end of the period, $(1 + i)^{-1}$ is used [14] where this value is called the discount factor. The discount factor can also be expressed as follows [10]:

$$v = \frac{1}{1+i} \tag{5}$$

2.3. Discrete Term Due Annuity

An Annuity Due is a series of payments paid at the beginning of a period. The value of an annuity of *m*-years with one unit payment at the beginning of the year for an insured aged x years can be expressed as follows [15]:

$$\ddot{a}_{x:\bar{m}|} = \sum_{k=0}^{m-1} v^k {}_k p_x \tag{6}$$

where,

 v^k : discount factor in the *k*-th period

 $_{k}p_{x}$: the probability that the insured who is now x years old will live for the next k years.

2.4. Term Life Insurance

Term life insurance is life insurance that protects the insured within a certain period of time. The actuarial present value can be expressed by the expected value of the random variable Z or E[Z] and can also be denoted by A. The actuarial present value of n-year term life insurance with a benefit payment of one unit can be expressed as follows [1]:

$$A_{x:\overline{n}|}^{1} = E[Z] = \sum_{k=0}^{n-1} (v^{k+1}) ({}_{k}p_{x})(q_{x+k})$$
(7)

where,

 v^{k+1} : discount factor at period (k+1)

 $_{k}p_{x}$: the probability that the insured who is now x years old will live for the next k years

 q_{x+k} : the probability that the insured will die at an age between (x+k) years and (x+k+1) years

2.5. Data Assumptions

In calculating premium reserves in this study, male and female life expectancy data from the Indonesian Mortality Table IV (2019) were used. The case study uses information from a male who is (x) 40 years old and realizes the importance of protecting himself from calamities or other unwanted events in the future. This makes him decide to register for life insurance with term life insurance. The coverage period (n) and the length of payment period (m) of the insurance chosen are for 20 years and 10 years respectively, with the amount of benefits (R) received upon death amounting to Rp100,000,000.00. Premium payments are made annually and at the beginning of each payment period. The assumptions used in the GPV method premium reserve calculation are as follows:

- 1. There are three types of expenses in life insurance companies, namely, initial fee (α) of Rp175,000, maintenance fee (β) of 3% of gross premium (*G*), and administrative fee (γ) of Rp360,000.
- 2. The initial fee (α) is paid once with the first premium payment, while the maintenance fee (β) and administration fee (γ) are paid every premium payment period until the payment period is completed,
- 3. The interest rate (*i*) used is the BI 7-Day Reverse Repo Rate (BI7DRR) set by Bank Indonesia on September 21, 2023 at 5.75%,

4. The insured will be given a benefit of *R* (Rupiah) in accordance with the company's provisions in the event of death within the agreed period with the company.

The calculation of premium reserves with this case study will be discussed by comparing several possible conditions, namely a) if the insured party is male and female with the same age (x), b) if the insured party is male with different ages (x) (30 years, 40 years, 50 years), and c) if the insured party is male, the same age (x) but with different interest rates (i) (4.75%; 5.75%; 6.75%).

2.6. Research Stages

In this research process, the first step that must be taken is to determine the data assumptions that will be used in the calculation. The assumptions required are determining the type of insurance used, determining the age of the insured (x), determining the length of the coverage period (n), determining the length of the premium payment period (m), determining the benefits of R (Rupiah) that will be received by the insured, and calculating the value of the mortality table based on the Indonesian Mortality Table IV (2019) and calculating the value of the discount factor with an interest rate of 5.75% based on the BI 7-Day Reverse Repo Rate (BI7DRR).

Next, determine the cash in and cash out of the term life insurance company and calculate the present value of the cash in and cash out. After that, determine the formula for calculating the gross premium value (*G*) and the formula for calculating the premium reserve (V_t). In the calculation of the premium reserve value, this research is carried out using the GPV method. After obtaining the formula for calculating premium reserves using the GPV method, calculations are carried out for case studies where in this calculation calculations are also carried out for the actuarial present value of term life insurance ($A_{x:\overline{nl}}^1$) and the cash value of discrete term life insurance annuities ($\ddot{a}_{x:\overline{ml}}$). Furthermore, the calculation of the insurance premium reserve value is carried out. In the calculation of premium reserves with the GPV method, there are several additional costs from insurance companies such as initial costs (α), maintenance costs (β), and administrative costs (γ).

The first step is to determine cash in and cash out, where cash in is all incoming costs or income of the insurance company such as gross premium (*G*) paid by the insured party to the company while cash out is all costs incurred by the insurance company for the benefit of the insurance policy. These costs can include initial costs (α), maintenance costs (β), and administrative costs (γ). Suppose cash out is denoted by Z_x and cash in is denoted by Y_x then,

$$Z_{x} = \begin{cases} 100.000.000. v^{k+1} + 175.000 + (0.03G + 360.000)\ddot{a}_{\overline{K+1}|}; K = 0, 1, ..., 19\\ 0; K \text{ others} \end{cases}$$
(8)

$$Y_{x} = \begin{cases} G. \ddot{a}_{\overline{K+1}}; K = 0, 1, 2, \dots, 9\\ 0; K \ others \end{cases}$$
(9)

Next is to determine the present value of cash out and cash in with the equivalence principle. According to Dickson et al (2013) this principle estimates the value of the gross premium set in such a way that the value of future gross losses is equal to zero and can be expressed as follows [6]:

$$E[PVBenefits] + E[PVExpenses] - E[PVGrossPremiumInflow] = 0$$
(10)

This value is obtained by the expected value of cash out and cash in which must be zero to fulfill the equivalence principle in equation (10) and can be formulated as follows based on equations (8) and (9):

$$E[Z_x] = \left(100.000.000 \cdot A^1_{40:\overline{20|}}\right) + 175.000 + (0.03G + 360.000)\ddot{a}_{40:\overline{10|}}$$
(11)

$$E[Y_x] = G.\ddot{a}_{40:\overline{10|}} \tag{12}$$

From the two expected values of cash out and cash in and the principle of equivalence in equation (10), it can be formulated for the gross premium value in this case study as follows:

$$G = \frac{\left(100.000.000 \cdot A_{40:\overline{20|}}^{1}\right) + 175.000 + (360.000 \cdot \ddot{a}_{40:\overline{10|}})}{\left(0.97.\ddot{a}_{40:\overline{10|}}\right)}$$
(13)

After obtaining the value of cash out, cash in, and gross premium, the GPV method formulation can be determined based on the general GPV formula according to Futami, namely [10]:

$$V_t = PVFCO_t - PVFCI_t \tag{14}$$

where,

 V_t : GPV method premium reserve value for the *t*-th year-end period

 $PVFCO_t$: the present value of all cash out components including costs and benefits, at the end of year t

 $PVFCI_t$: the present value of all components of cash in earned from gross premium income, at the end of year t

So it can be formulated the value of premium reserves with the GPV method by substituting equations (11) and (12) into equation (14) as follows:

$$V_0 = \left(100.000.000 \cdot A_{40:\overline{10|}}^1\right) + 175.000 + \left[(0,03G + 360.000)\ddot{a}_{40:\overline{10|}}\right] - G \cdot \ddot{a}_{40:\overline{10|}}$$
(15)

$$V_{t} = \left(100.000.000 \cdot A_{40+t:\overline{20-t|}}^{1}\right) + \left[(0,03G + 360.000)\ddot{a}_{40+t:\overline{10-t|}}\right] - G \cdot \ddot{a}_{40+t:\overline{10-t|}}; \ 1 \le t \le 10 \ (16)$$
$$V_{t} = 100.000.000 \cdot A_{40+t:\overline{20-t|}}^{1}; \ 10 < t \le 20$$
(17)

where:

t	:	years that have elapsed since the policy commenced
G	:	gross premium
$\ddot{a}_{40:\overline{10 }}$:	cash value of a 10-year term annuity with one unit payment at the beginning of the year for a 40-year-old insured person
$A^1_{40:\overline{20 }}$:	actuarial present value of 20-year term life insurance of a person aged 40 years until the insurance coverage period ends.
$\ddot{a}_{40+t:\overline{10-t} }$:	cash value of a $(10-t)$ year term annuity with one unit payment at the beginning of the year for an insured aged $(40+t)$ year
$A^1_{40+t:\overline{20-t }}$:	actuarial present value of $(20-t)$ year term life insurance of a person aged $(40+t)$ years until the insurance coverage period ends

3. RESULTS AND DISCUSSION

After obtaining the formulation of the GPV method premium reserve value, the premium reserve value for the three conditions can be found using equations (15), (16), and (17) so that the results of the premium reserve value for male and female insureds of the same age are as follows:

Table 1.	GPV Method	Premium Reserve	Value for Male an	d Female Insureds
----------	-------------------	------------------------	-------------------	-------------------

U	Gender	
V_t –	Male	Female
0	Rp 0	Rp 0
1	Rp 344,578	Rp 147,670
2	Rp 875,124	Rp 479,333
3	Rp 1,414,543	Rp 817,605
4	Rp 1,961,651	Rp 1,162,967
5	Rp 2,513,275	Rp 1,513,954
6	Rp 3,067,095	Rp 1,868,058
7	Rp 3,619,732	Rp 2,221,673
8	Rp 4,168,626	Rp 2,575,902
9	Rp 4,712,063	Rp 2,928,985
10	Rp 5,248,245	Rp 3,280,045

V	Gender	
V_t –	Male	Female
11	Rp 5,067,763	Rp 3,173,326
12	Rp 4,830,014	Rp 3,030,946
13	Rp 4,526305	Rp 2,847,705
14	Rp 4,147.230	Rp 2,619,003
15	Rp 3,685,489	Rp 2,337,929
16	Rp 3,133,125	Rp 1,999,016
17	Rp 2,487,347	Rp 1,598,334
18	Rp 1,748,067	Rp 1,133,621
19	Rp 918,203	Rp 601,418
20	Rp 0	Rp 0

Table 1 shows that the reserve value using the GPV method increases until the last year of the payment period and decreases after the payment period is completed until the last year of coverage. The premium reserve value for male policies is greater than for female policies at the age of 40 years. This is due to the greater mortality rate for men than women based on the Indonesian Mortality Table IV (2019). Furthermore, the premium reserve value for male insureds with different ages is as follows:

V _	Age (x)			
V_t –	x = 30	x = 40	x = 50	
0	Rp 0	R p 0	Rp 0	
1	Rp 26,984	Rp 344,578	Rp 648,758	
2	Rp 234,753	Rp 875,124	Rp 1,476,797	
3	Rp 448,668	Rp 1,414,543	Rp 2,305,276	
4	Rp 669,115	Rp 1,961,651	Rp 3,130,232	
5	Rp 896,504	Rp 2,513,275	Rp 3,950,465	
6	Rp 1,129,287	Rp 3,067,095	Rp 4,764,735	
7	Rp 1,366,834	Rp 3,619,732	Rp 5,577,473	
8	Rp 1,607,495	Rp 4,168,626	Rp 6,396,139	
9	Rp 1,850,525	Rp 4,712,063	Rp 7,231,344	
10	Rp 2,092,199	Rp 5,248,245	Rp 8,093,256	
11	Rp 2043,035	Rp 5,067,763	Rp 7,635,901	
12	Rp 1,971,314	Rp 4,830,014	Rp 7,123,914	
13	Rp 1,872,710	Rp 4,526,305	Rp 6,556,116	
14	Rp 1,743,593	Rp 4,147,230	Rp 5,925,555	
15	Rp 1,578,110	Rp 3,685,489	Rp 5,219,902	
16	Rp 1,370,992	Rp 3,133,125	Rp 4,424,754	
17	Rp 1.115,595	Rp 2,487,347	Rp 3,522,411	
18	Rp 805,779	Rp 1,748,067	Rp 2,496,405	
19	Rp 435,934	Rp 918,203	Rp 1,328,605	
20	Rp 0	Rp 0	Rp 0	

Table 2. GPV Method Premium Reserve Value for Male Insured with Different Age

Table 2 shows that the value of reserves using the GPV method for male-insured policies increases with age until the last year of the payment period, then decreases after the payment period is completed until the last year of coverage. The greater the age of the insured, the greater the premium reserve value generated. This is due to the higher mortality rate as a person gets older. Then, the premium reserve value for the insured male with the same age but different rates is as follows:

Table 3. GPV Method Premium Reserve Value for Male Insured with Different Interest Rates

V	I	Rate of Interest (i)	
V _t	<i>i</i> = 4.75%	<i>i</i> = 5.75%	<i>i</i> = 6.75%
0	Rp 0	Rp 0	Rp 0
1	Rp 389,425	Rp 344,578	Rp 304,126
2	Rp 961,843	Rp 875,124	Rp 796,604
3	Rp 1,539,921	Rp 1,414,543	Rp 1,300,597
4	Rp 2,122,245	Rp 1,961,651	Rp 1,815,174

17	Rate of Interest (i)			
V_t -	<i>i</i> = 4.75%	<i>i</i> = 5.75%	<i>i</i> = 6.75%	
5	Rp 2,705,420	Rp 2,513,275	Rp 2,337,422	
6	Rp 3,286,918	Rp 3,067,095	Rp 2,865,263	
7	Rp 3,863,168	Rp 3,619,732	Rp 3,395,558	
8	Rp 4,431,439	Rp 4,168,626	Rp 3,925,971	
9	Rp 4,989,854	Rp 4,712,063	Rp 4,455,009	
10	Rp 5,536,459	Rp 5,248,245	Rp 4,981,097	
11	Rp 5,318,458	Rp 5,067,763	Rp 4,833,877	
12	Rp 5,043,125	Rp 4,830,014	Rp 4,629,906	
13	Rp 4,702,310	Rp 4,526,305	Rp 4,359,977	
14	Rp 4,287,266	Rp 4,147,230	Rp 4,014,049	
15	Rp 3,791,475	Rp 3,685,489	Rp 3,584,053	
16	Rp 3,207,880	Rp 3,133,125	Rp 3,061,129	
17	Rp 2,534,724	Rp 2,487,347	Rp 2,441,435	
18	Rp 1,773,045	Rp 1,748,067	Rp 1,723,710	
19	Rp 926,969	Rp 918,203	Rp 909,602	
20	- Rp 0	Rp 0	Rp 0	

Table 3 shows that the reserve value using the GPV method increases until the last year of the payment period and decreases after the payment year is completed until the last year of the coverage period. The premium reserve value gets smaller with higher interest rates, while the premium reserve value gets bigger with lower interest rates. This is due to the interest rate, a variable or variable discount factor.

4. CONCLUSION

Based on the results and discussion that has been explained, it can be concluded that the premium reserve formula with the Gross Premium Valuation method of term life insurance in this study is as follows:

$$V_{t} = \begin{cases} 0; t = 0\\ (100.000.000 \cdot A_{40+t:\overline{20-t}|}^{1}) + [(0,03G + 360.000)\ddot{a}_{40+t:\overline{10-t}|}] - G \cdot \ddot{a}_{40+t:\overline{10-t}|}; 1 \le t \le 10\\ 100.000.000 \cdot A_{40+t:\overline{20-t}|}^{1}; 10 < t \le 20 \end{cases}$$

It was also found that the premium reserve with the GPV method at the beginning of the first year of the policy is zero, increases until the last year of the payment period, and then decreases after the payment period is completed until the last year of coverage. The premium reserve for the male insured is greater than the female insured for the same age. If the insured is male and age varies, the required premium reserve is greater if the insured is older, and if the insured is male and age varies, the required premium reserve is less for higher interest rates.

REFERENCES

- [1] A. R. Effendie, *Matematika Aktuaria Dengan Software R*, Yogyakarta. Gadjah Mada University Press, 2015.
- [2] S. Artika, I. G. P. Purnaba, dan D. C. Lesmana, "Penentuan Premi Asuransi Jiwa Berjangka Menggunakan Model Vasicek Dan Model Cox-Ingersoll-Ross (CIR)," Hlm. 129–139, 2018.
- [3] R. E. Larson dan E. A. Gaummnitz, *Life Insurance Mathematics*. 1951.
- [4] W. Ariani, N. Satyahadewi, dan H. Perdana, "Penentuan Cadangan Premi Pada Asuransi Jiwa Dwiguna Joint Life Dengan Metode Premium Sufficiency," *Buletin Ilmiah Math. Stat Dan Terapannya (Bimaster)*, Vol. 09, No. 1, Hlm. 205–212, 2020.
- [5] T. Futami, *Matematika Asuransi Jiwa, Bagian II*. Tokyo: Incorporated Foundation Oriental Life Insurance Cultural Development Center, 1994.

- [6] D. C. M. Dickson, M. R. Hardy, dan H. R. Waters, *Actuarial Mathematics For Life Contingent Risks*. 2013. [Daring]. Tersedia Pada: Www.Cambridge.Org/Statistics.
- [7] Y. Hikmah dan H. H. Khuzaimah, "Perhitungan Cadangan Premi Asuransi Jiwa Dengan Metode Gross Premium Valuation (GPV)," Vol. 1, Hlm. 61–69, 2019.
- [8] N. Hasnah, "Kajian Metode Commissioners, Illinois Dan Canadian Dalam Menentukan Cadangan Pada Asuransi Jiwa Dwiguna," *Jurnal Matematika Unand*, Vol. 4, No. 4, Hlm. 99–106, 2019.
- [9] E. A. Prionggo, M. N. Pratama, dan F. Indrayatna, "Estimation Of Prospective Benefit Reserve Based On Gross Premium Valuation Method Using Indonesian Mortality Table Iv And De-Moivre Assumptions," *Enthusiastic International Journal Of Statistics And Data Science*, Vol. 2, No. 2, Hlm. 56–67, 2022, [Daring]. Tersedia Pada: Https://Journal.Uii.Ac.Id/Enthusiastic
- [10] T. Futami, *Matematika Asuransi Jiwa, Bagian I.* Tokyo: Incorporated Foundation Oriental Life Insurance Cultural Development Center, 1993.
- [11] S. Pratiwi, N. Satyahadewi, dan H. Perdana, "Penentuan Cadangan Premi Asuransi Jiwa Dwiguna Joint Life Dengan Metode Canadian," *Buletin Ilmiah Math. Stat Dan Terapannya (Bimaster)*, Vol. 11, No. 2, Hlm. 239–246, 2022.
- [12] D. Eurico, S. Kezia, L. Noviyanti, dan A. Z. Soleh, "Cadangan Prospektif Produk Asuransi Jiwa Endowment Dengan Metode Gross Premium Valuation," *Jurnal Matematika Integratif*, Vol. 17, No. 2, Hlm. 97, Jan 2022, Doi: 10.24198/Jmi.V17.N2.34360.97-108.
- [13] M. Subhan, *Pengantar Matematika Aktuaria*. Padang, 2019.
- [14] S. G. Kellison, *The Theory Of Interest*, 2 Ed. Sydney, 1991.
- [15] N. L. Bowers, H. U. Geerber, J. C. Hickman, D. A. Jones, dan C. J. Nesbitt, *Actuarial Mathematics*. 1997, 1997. [Daring]. Tersedia Pada: Http://Eduktodos.Dyndns.Org