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H_{∞} state feedback for linear systems with decentralized control inputs

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Abstract

This paper considers H_∞ state feedback with decentralized structure for interconnected systems. The connection between subsystems is described by a directed graph. To design a decentralized H_∞ controller, we use the information from its own subsystem and other subsystems based on the interconnection. Decentralized controller is defined as a solution of bilinear matrix inequality (BMI) problem, which is then solved by using the homotopy approach. Two numerical examples are performed to show validity of the design procedure.

Keywords: H_∞ state feedback, decentralized controller, homotopy method, bilinear matrix inequality (BMI), linear matrix inequality (LMI).

The invented contribution: A newly smart design of a decentralized H_∞ controller with numerical examples for the validity of the design procedure was invented by using homotopy method with the information from its own subsystem and other subsystems based on the interconnection.

ARTICLES

I. Introduction

In recent years, many engineering systems consist of the interconnection of systems. One can develop system model by modelling the individual subsystem and then connect these models together. For example, an aircraft hydraulic system consists of many individual components each with their own inputs and outputs by making models of the components and then connecting the models together. The interconnected system makes the chance of system overload reduced.

In [1], the control method is divided into two categories, i.e., centralized and decentralized controller. The centralized controller is a single control strategy. If the

system to be designed for control is a large scale system, the centralized control is not efficient. If the centralized controller is damaged, the whole system will be uncontrolled automatically. For large scale system, decentralized controller is one way to control the system efficiently. In this sense, it is necessary to decompose the large scale system into a number of interconnected subsystems [2].

In [3], the authors propose an approach to design a decentralized control using a solution of linear matrix inequality (LMI). The approach requires that the input matrix of each subsystem is invertible, i.e. the number of independent control inputs is the same with the number of state variables in each subsystem. The authors in [4] design a decentralized output feedback controller for large-scale systems using LMI where the solution does not require

invertibility of the input matrix of each subsystem. In [5], decentralized and semi-decentralized state-feedback H_∞ controllers, which only require local state information, can be efficiently computed using the powerful linear matrix inequality (LMI) solvers. In [6], the authors design a decentralized H_∞ controller by using homotopy method to find a feasible solution of bilinear matrix inequality (BMI) problems. Moreover, the authors in [7] design a robust decentralized H_∞ controller for uncertain large scale interconnected systems using homotopy method.

In this paper, we design a decentralized H_∞ controller for interconnected systems. The connection between subsystems is described by a directed graph. Each subsystem is represented by a node in the graph. To design a decentralized H_∞ controller, we use the information from its own subsystem and other subsystems based on the interconnection. Because of that, the structure of decentralized controller is based on the interconnection of systems. Our approach to obtain decentralized H_∞ controller is similar to that of [6]. More precisely, we first consider a centralized controller, i.e., H_∞ state feedback based on LMI [8], then deform the centralized H_∞ controller to decentralized H_∞ controller by using the homotopy method. Two numerical examples are presented to show validity of the design procedure.

Notations: In standard matrix notation, each element of R^n is typically written as $n \times 1$ column vectors and sometimes as $1 \times n$ row vector. A matrix $X \in R^{m \times n}$ is a rectangular array of real numbers with m rows and n columns. Let X be a matrix with size $m \times n$, the transpose matrix of X is written as X^T with size $n \times m$.

The notation of $\text{diag}\{X_1, X_2, \dots, X_n\}$ denotes a block diagonal matrix whose diagonal blocks are $X_1, X_2, \dots, X_n$ in order. For a symmetric matrix X , we use $X > 0$ ($X < 0$) to mean that it is symmetric positive (negative) definite.

The H_∞ norm of a transfer function matrix $G(s)$ is defined as [9].

$$\|G(s)\|_\infty = \max_{\omega} \bar{\sigma}(G(j\omega))$$

where $\bar{\sigma}$ is equal to maximum singular value. Generally, the system $G(s)$ is selected to be a transfer function between exogenous inputs (commands, disturbance, and noise) and exogenous output (error signals to be minimize).

Hadamard product is the element-wise multiplication which takes two matrices of the same dimensions and produces another matrix of the same dimension as the operands. The Hadamard product is commutative, associative and distributive over addition:

$$U \odot V = V \odot U,$$

$$U \odot (V \odot W) = (U \odot V) \odot W,$$

$$U \odot (V + W) = U \odot V + U \odot W,$$

$$(kU) \odot V = U \odot (kV) = k(U \odot V),$$

$$U \odot 0 = 0 \odot U = 0,$$

where $U, V, W, 0$ are matrices of the same dimension, $k \in \mathbb{R}$ and 0 is a matrix whose elements are all zeros.

II. Problem Formulation

Consider a large-scale system consisting of N subsystems described by

$$\dot{x}_i = \sum_{j=1}^N A_{ij}x_j + B_{1i}w_i + B_{2i}u_i, \quad (1)$$

$$z_i = C_i x_i + D_i w_i,$$

where $x_i \in \mathbb{R}^{n_i}$ is the state vector, $w_i \in \mathbb{R}^{r_i}$ is the disturbance, $u_i \in \mathbb{R}^{m_i}$ is the control input, and $z_i \in \mathbb{R}^{p_i}$ is the performance output of subsystem i ($i = 1, 2, \dots, N$). A_{ij} , B_{1i} , B_{2i} , C_i , and D_i are constant matrices of the i -th subsystem with appropriate sizes.

For system (1), we assume that the connection among subsystems is described by a directed graph. Each subsystem is represented by a node in the graph. There is an edge from j to i if the controller in the i -th subsystem can use the information from the j -th subsystem. There is a self loop in every node because the controller in each subsystem can use the information from its own subsystem. The directed graph can be represented as a matrix $\mathcal{L} = [\ell_{ij}] \in \{0, 1\}^{N \times N}$, where $\ell_{ij} = 1$

if there is an edge from j -th node to i -th node, and $\ell_{ij} = 0$ otherwise.

We consider the following static state-feedback control law

$$u_i = \sum_{j=1}^N K_{ij} \ell_{ij} x_j, \quad (2)$$

where $K_{ij} \in \mathbb{R}^{m_i \times n_j}$ are gain matrices to be determined.

The problem is to find the controller (2) stabilizing system (1) such that the influence of the disturbance w on the controlled output z is attenuated to a specified level. For the whole system, we define the disturbance and the performance output by

$$w = [w_1^T \ w_2^T \ \dots \ w_N^T]^T \in \mathbb{R}^r, r = r_1 + \dots + r_N,$$

$$z = [z_1^T \ z_2^T \ \dots \ z_N^T]^T \in \mathbb{R}^p, p = p_1 + \dots + p_N,$$

and the transfer function from w to z of the closed-loop system is denoted by $T_{zw}(s)$. The control problem discussed in this paper is formulated as follows:

Decentralized H_∞ control problem. For a given disturbance attenuation level $\gamma > 0$, design a decentralized controller (2) for system (1) such that the closed-loop system is stable and $\|T_{zw}(s)\|_\infty < \gamma$.

We assume that there exists a centralized H_∞ controller with the same disturbance attenuation level γ , because the decentralized controllers cannot achieve better performance than the best centralized controller.

III. Existence Condition for a Decentralized H_∞ Controller

In this section, we summarize the existence condition for a decentralized H_∞ controller. First of all, we formulate the closed-loop system. The closed-loop system obtained by substituting the control law (2) to the system (1) is as follows

$$\dot{x}_i = \sum_{j=1}^N A_{ij}x_j + B_{1i}w_i + B_{2i} \sum_{j=1}^N K_{ij}\ell_{ij}x_j, \quad (3)$$

$$z_i = C_i x_i + D_i w_i.$$

Observe that there is state, disturbance, performance output and several feedback gains in every subsystem. We collect the state x_i and the interconnection coefficient ℓ_{ij} as

$$x = [x_1^T \ x_2^T \ \dots \ x_N^T]^T \in R^n, n = n_1 + \dots + n_N,$$

$$L_D = \begin{bmatrix} \ell_{11}J_{m_1,n_1} & \ell_{12}J_{m_1,n_2} & \dots & \ell_{1N}J_{m_1,n_N} \\ \ell_{21}J_{m_2,n_1} & \ell_{22}J_{m_2,n_2} & \dots & \ell_{2N}J_{m_2,n_N} \\ \vdots & \vdots & \ddots & \vdots \\ \ell_{N1}J_{m_N,n_1} & \ell_{N2}J_{m_N,n_2} & \dots & \ell_{NN}J_{m_N,n_N} \end{bmatrix}, \quad (4)$$

where $J_{m,n}$ denotes a matrix of size $m \times n$ whose elements are all 1.

In order to summarize the closed-loop system (3) by using (4), we define the following matrices

$$A = \begin{bmatrix} A_{11} & A_{12} & \dots & A_{1N} \\ A_{21} & A_{22} & \dots & A_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ A_{N1} & A_{N2} & \dots & A_{NN} \end{bmatrix} \in R^{n \times n},$$

$$B_1 = \text{diag}\{B_{11}, B_{12}, \dots, B_{1N}\} \in R^{n \times r},$$

$$B_2 = \text{diag}\{B_{21}, B_{22}, \dots, B_{2N}\} \in R^{n \times m}$$

$$C = \text{diag}\{C_1, C_2, \dots, C_N\} \in R^{p \times n},$$

$$D = \text{diag}\{D_1, D_2, \dots, D_N\} \in R^{p \times r}. \quad (5)$$

Finally, by using (4) and (5), the closed-loop system (3) can be written in a compact form as

$$\dot{x} = (A + B_2(L_D \odot K))x + B_1w, \quad (6)$$

$$z = Cx + Dw,$$

where $\odot$ is the Hadamard product operator. The decentralized controller gain matrix K_D is defined as $L_D \odot K$. In the above formulation (6), the unknown quantity is matrix K , whereas the other matrices are derived from the system (1).

In order to formulate the existence condition of decentralized H_∞ controller, we use the following fundamental result of H_∞ control [8-10].

Lemma 3.1 The following statements are equivalent:

- A is a stable matrix and $\|C(sI - A)^{-1}B + D\|_\infty < \gamma$.
- There exists a matrix $P > 0$ such that

$$\begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & -\gamma I & D^T \\ C & D & -\gamma I \end{bmatrix} < 0.$$

Next, we apply Lemma 3.1 to (6). With the definition

$$F(L, K, P) = \begin{bmatrix} A^T P + PA & PB_1 & C^T \\ B_1^T P & -\gamma I_r & D^T \\ C & D & -\gamma I_p \end{bmatrix} + \begin{bmatrix} PB_2 \\ 0_{r \times m} \\ 0_{p \times m} \end{bmatrix} (L \odot K) \begin{bmatrix} I_n & 0_{n \times r} & 0_{n \times p} \end{bmatrix} + \left\{ \begin{bmatrix} PB_2 \\ 0_{r \times m} \\ 0_{p \times m} \end{bmatrix} (L \odot K) \begin{bmatrix} I_n & 0_{n \times r} & 0_{n \times p} \end{bmatrix} \right\}^T,$$

we state the following result:

Theorem 3.1 *System (1) is stabilizable with the disturbance attenuation level γ by a decentralized controller (2) if and only if there exist a matrix K and a matrix $P > 0$ such that*

$$F(L_D, K, P) < 0. \quad (7)$$

Observe that (7) is a Bilinear Matrix Inequality (BMI). In order to compute the decentralized controller, we use the homotopy method, originally proposed in [6]. The first step of homotopy method is computing the centralized H_∞ controller.

IV. Computation of Centralized Controller

This section describes a method to compute the centralized H_∞ controller. In centralized H_∞ controller, each subsystem can use the information from all subsystems, i.e. $\mathcal{L} = J_{N,N}$ and $L_F = J_{m,n}$. It follows that the

centralized controller K_F is the solution of the following **BMI**

$$\begin{bmatrix} A^T P + K_F^T B_2^T P + PA + PB_2 K_F & PB_1 & C^T \\ B_1^T P & -\gamma I_r & D^T \\ C & D & -\gamma I_p \end{bmatrix} < 0. \quad (8)$$

The **BMI** in (8) can be transformed into an LMI by multiplying the above matrix from the left and right by $\text{diag}\{P^{-1}, I, I\}$. When we introduce $Q = P^{-1}$ and $M = K_F Q$, the following LMI is obtained

$$\begin{bmatrix} QA^T + M^T B_2^T + AQ + B_2 M & B_1 & QC^T \\ B_1^T & -\gamma I_r & D^T \\ CQ & D & -\gamma I_p \end{bmatrix} < 0,$$

where $Q > 0$. The centralized controller K_F and matrix P are obtained by using $K_F = M Q^{-1}$ and $P = Q^{-1}$, respectively.

One way of generating a centralized H_∞ controller randomly is solving the following LMI

$$\begin{bmatrix} QA^T + M^T B_2^T + AQ + B_2 M & B_1 & QC^T \\ B_1^T & -\gamma I_r & D^T \\ CQ & D & -\gamma I_p \end{bmatrix} < -\Phi, \quad (9)$$

where Φ is a small positive-definite matrix.

V. Computation of Decentralized Controller

In this section, we discuss the homotopy method to find the decentralized controller. In

order to use the homotopy method, we introduce the following matrix function:

$$H(K, P, \lambda) = F((1 - \lambda)L_F + \lambda L_D, K, P),$$

where $\lambda \in [0, 1]$ is a real number and $L_F = J_{m,n}$ represents the interconnection of centralized H_∞ controller. To construct the homotopy path, we define $\lambda_k = k/\mu$, where μ is a positive integer and $k = 0, 1, 2, \dots, \mu$. If the following problem

$$H(K, P, \lambda_k) < 0$$

is feasible, we denote the solution by (K_k, P_k) . The algorithm for computing decentralized H_∞ controller is as follows.

```

1: Initialize maxIter to a positive integer, e.g.
   maxIter := 50
2: Set iter := 1
3: while iter ≤ maxIter do
4:   Generate a small positive-definite
     matrix  $\Phi$  randomly
5:   Compute centralized  $H_\infty$  controller  $K_F$ 
     and matrix  $P$  using (9)
6:   Set  $k := 0, K_0 := K_F$  and  $P_0 := P$ 
7:   Initialize  $\mu$  to a positive integer and its
     upper bound  $\mu_{max}$ , e.g.  $\mu := 2$  and
      $\mu_{max} := 2^{10}$ 
8:   while  $k < \mu$  and  $\mu \leq \mu_{max}$  do
9:     Set  $\lambda_{k+1} := (k + 1)/\mu$ 
10:    Compute a solution  $K$  of LMI
        $H(K, P_k, \lambda_{k+1}) < 0$ 
11:    If step 6 is feasible then
12:      Set  $K_{k+1} := K$ 
13:      Compute a solution  $P$  of

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       LMI  $H(K_{k+1}, P, \lambda_{k+1}) < 0$ ,
       then set  $P_{k+1} := P$ 
14:     Set  $k := k + 1$ 
15:   else
16:     Compute a solution  $P$  of
       LMI  $H(K_k, P, \lambda_{k+1}) < 0$ 
17:     if step 12 is feasible then
18:       Set  $P_{k+1} := P$ 
19:       Compute a solution  $K$ 
          $H(K, P_{k+1}, \lambda_{k+1}) < 0$ ,
         then  $K_{k+1} := K$ 
20:       Set  $k := k + 1$ 
21:     else
22:       Set  $\mu := 2\mu$ ,
          $P_{2k} := P_k$  ,
          $K_{2k} := K_k$ 
23:       Set  $k := 2k$ 
24:     end if
25:   end while
26: end while
27: if  $k = \mu$  then
28:   The pair  $(K_\mu, P_\mu)$  is a solution of
     the BMI
29: else
30:   Set iter := iter + 1
31: end if
32: end while

```

VI. Numerical Example 1

In this section, we present an example to demonstrate the homotopy method based algorithm to compute the decentralized controller. Consider a system consisting of four subsystems, where the connection among four

subsystems is described by directed graph in **Fig. 1**.

Figure 1. Illustration of interconnection of four subsystems.

The interconnection is represented as

$$\mathcal{L} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}.$$

We combine the matrix as in (5), and the coefficient matrices are

$$A_{11} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & -1 & -2 \\ 1 & -1 & 1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}, \quad A_{14} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_{21} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -1 & -2 \\ 4 & 2 \end{bmatrix},$$

$$A_{23} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad A_{24} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$

$$A_{31} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad A_{32} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix},$$

$$A_{33} = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix}, \quad A_{34} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

$$A_{41} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_{42} = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix},$$

$$A_{43} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_{44} = \begin{bmatrix} -2 & 2 \\ 3 & 1 \end{bmatrix},$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix}, \quad (10)$$

$$K_F = \begin{bmatrix} 24.45 & -12.39 & 29.21 & -12.03 & 6.33 & 3.84 & 6.06 & -0.00 & 1.14 \\ -7.72 & 1.80 & -10.97 & -8.42 & -3.62 & 4.41 & -0.68 & -0.20 & 5.14 \\ -13.41 & 0.85 & -15.26 & 1.22 & -5.46 & 20.48 & -7.63 & -0.60 & 0.71 \\ -113.56 & -25.36 & -120.29 & -67.86 & -70.58 & -71.72 & -51.26 & -10.20 & -11.79 \end{bmatrix} \quad (15)$$

$$K_D = \begin{bmatrix} 290.63 & -18.80 & 514.83 & 47.59 & -30.18 & 0 & 0 & 0 & 0 \\ -40.44 & 7.49 & -71.07 & -35.11 & 14.55 & 0 & 0 & 0 & 0 \\ -244.66 & 48.30 & -105.71 & 73.36 & -202.69 & -683.87 & -121.09 & 0 & 0 \\ 0 & 0 & 0 & -347.56 & -292.63 & 242.15 & -19.15 & -62.51 & 104.54 \end{bmatrix} \quad (16)$$

$$B_1 = \text{diag} \left\{ \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}, \quad (11)$$

$$B_2 = \text{diag} \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}, \quad (12)$$

$$C = \text{diag} \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \right\}, \quad (13)$$

$$D = \text{diag}\{[1], [1], [1], [1]\}. \quad (14)$$

The state matrix A in (10) is unstable because some eigenvalues have positive real parts. We obtain the centralized H_∞ controller in (15). One can check that the closed-loop system is asymptotically stable by showing that $A + B_2 K_F$ is Hurwitz.

For system (10)-(14), the minimum H_∞ disturbance attenuation level achieved by a centralized controller is **1.675**.

When we use the homotopy method to compute the decentralized controller, and use $\gamma = 2.675$ as the desired disturbance attenuation level. **Equation (16)** is the decentralized controller K_D .

According to (6), the state matrix of the closed-loop system by using decentralized H_∞ controller is $A + B_2 K_D$. Since $A + B_2 K_D$ is Hurwitz, the closed-loop system is asymptotically stable. The H_∞ disturbance attenuation level achieved by decentralized H_∞ controller (16) is **1.7915**.

VII. Numerical Example 2

In this section, we apply the proposed method to five storey building systems. Each storey building is defined as a subsystem, which is represented by a node. The connection among five storey buildings is described by the directed graph depicted in **Fig. 2**.

Figure 2. Illustration of interconnection in the storey building systems.

The equation of motion for i -th subsystem is the second-order differential equation

$$M_i \ddot{x}(t) + C_i \dot{x}(t) + K_i x(t) = E_i w(t) + G_i u(t), \quad (17)$$

where M_i , K_i , and C_i are the mass, spring, and damper coefficients of the i -th subsystem respectively, u is a force vector, and w is an external interference. Matrices E_i and G_i represent external disturbance and control force coefficients, respectively. The parameter values in storey building systems shown in **Table 1**.

Table 1. Parameter values for five storey building systems [11].

i	M_i	K_i	C_i	G_i	E_i
1	215200	147×10^6	171800	-10^3	10^3
2	209200	113×10^6	182000	-10^3	10^3
3	207000	99×10^6	202500	-10^3	10^3
4	204800	89×10^6	231100	-10^3	10^3
5	266100	84×10^6	251100	-10^3	10^3

To write the state-space equation for i -th subsystem, we introduce

$$x_1 = x \quad \text{and} \quad x_2 = \dot{x}$$

which represents the interstory drifts $x(t)$ and velocities $\dot{x}(t)$. By substituting the above equations into (17), we obtain the following system:

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = -M_i^{-1}K_i x_1 - M_i^{-1}C_i x_2 + M_i^{-1}E_i w + M_i^{-1}G_i u.$$

The five storey building systems can be written as (1), where

$$A_i = \begin{bmatrix} 0 & 1 \\ -M_i^{-1}K_i & -M_i^{-1}C_i \end{bmatrix},$$

$$B_{1i} = \begin{bmatrix} 0 \\ M_i^{-1}E_i \end{bmatrix}, \quad B_{2i} = \begin{bmatrix} 0 \\ M_i^{-1}G_i \end{bmatrix},$$

$$C_i = [0 \quad 1], \quad D_i = 1.$$

The performance output of i -th subsystem is defined as

$$z = C_i x + D_i w,$$

and the interconnection is represented as

$$\mathcal{L} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

The coefficient matrix in this example are

$$A = \text{diag}$$

$$\left\{ \begin{bmatrix} 0 & 1 \\ -683.08 & -0.7983 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -540.15 & -0.8699 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 & 1 \\ -478.26 & -0.9782 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -434.57 & -1.128 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 & 1 \\ -315.67 & -0.9436 \end{bmatrix} \right\},$$

$$K_F = 10^5 \times$$

$$\begin{bmatrix} -1.4808 & 0.0067 & -0.0090 & -0.0077 & -0.0096 & -0.0374 & -0.0082 & -0.0263 & -0.0060 & -0.0114 \\ 0.0001 & 0.0073 & -1.1286 & 0.0080 & -0.0007 & -0.0102 & 0.0003 & -0.0039 & -0.0004 & -0.0052 \\ 0.0118 & 0.0371 & 0.0082 & 0.0115 & -0.9813 & 0.0085 & 0.0066 & -0.0053 & 0.0050 & 0.0026 \\ 0.0094 & 0.0260 & 0.0065 & 0.0054 & 0.0069 & 0.0077 & -0.841 & 0.0077 & 0.0036 & 0.0035 \\ 0.0066 & 0.0149 & 0.0058 & 0.0080 & 0.0046 & -0.0010 & 0.0045 & 0.0064 & -0.8361 & 0.0099 \end{bmatrix} \quad (18)$$

$$K_D$$

$$= \begin{bmatrix} -0.89 & 528.62 & 4.52 & -5.05 & 0 & 0 & 0 & 0 & 0 & 0 \\ -5.98 & -2.14 & -1.78 & 527.92 & 0.80 & -0.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.59 & -0.11 & -2.05 & 524.91 & -0.14 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.01 & 0.04 & -3.1 & 520.09 & 0.79 & -0.42 \\ 0 & 0 & 0 & 0 & 0 & 0 & -0.47 & -0.29 & -2.24 & 517.16 \end{bmatrix} \quad (19)$$

$$B_1 = \text{diag} \left\{ \begin{bmatrix} 0 \\ 0.0046 \end{bmatrix}, \begin{bmatrix} 0 \\ 0.0047 \end{bmatrix}, \begin{bmatrix} 0 \\ 0.0048 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 \\ 0.0048 \end{bmatrix}, \begin{bmatrix} 0 \\ 0.0037 \end{bmatrix} \right\},$$

$$B_2 = \text{diag} \left\{ \begin{bmatrix} 0 \\ -0.0046 \end{bmatrix}, \begin{bmatrix} 0 \\ -0.0047 \end{bmatrix}, \begin{bmatrix} 0 \\ -0.0048 \end{bmatrix}, \right. \\ \left. \begin{bmatrix} 0 \\ -0.0048 \end{bmatrix}, \begin{bmatrix} 0 \\ -0.0037 \end{bmatrix} \right\},$$

$$C = \text{diag} \left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \end{bmatrix}^T \right\}$$

$$D = \text{diag}\{[1], [1], [1], [1]\}$$

We obtain the centralized H_∞ controller in (18). One can check that the closed-loop system is asymptotically stable by showing that $A + B_2 K_F$ is Hurwitz. In this example, matrix $A + B_2 K_F$ is Hurwitz, i.e. all eigenvalues have negative real parts.

For this system, the H_∞ disturbance attenuation level achieved by a centralized controller is **1.0010**.

When we use the homotopy method to compute the decentralized controller, and use $\gamma = 1.150$ as the desired disturbance attenuation level. The homotopy method produces the matrix K_D in (19).

According to (6), the state matrix of the closed-loop system by using decentralized H_∞ controller is $A + B_2 K_D$. Since $A + B_2 K_D$ is Hurwitz, the closed-loop system is asymptotically stable. The H_∞ disturbance attenuation level achieved by decentralized H_∞ controller is 1.0014.

VIII. Conclusions

This paper is concerned with the static state feedback with decentralized structure for interconnected systems using homotopy method. The connection between subsystems is represented as a directed graph, which represents that a subsystem can use other subsystem information. There is a self loop in every node because the controller in each subsystem can use the information from its own subsystem. Based on the simulation results, the decentralized H_∞ controller gives a pretty good disturbance attenuation level compared with the centralized H_∞ state feedback controller.

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